COMITATO NAZIONALE PER L'ENERGIA NUCLEARE
Laboratori Nazionali di Frascati

LNF - 66/67
12 Dicembre 1966

C. Mencuccini and A. Reale: REMARKS ON THE η MESON CUSP EFFECT IN π^0 PHOTOPRODUCTION. -

(Nota interna: n. 342)

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In a recent past the knowledge of the πN system in the energy region of the second resonance (N^{**}, 1518 MeV) has been considerably improved, essentially due to some accurate phase shift analyses of the πN scattering[1 ÷ 5].

It turns out that in this energy region strong inelastic effects mainly in P_{11}, S_{11} and S_{31} states occur, superimposed to the resonant behaviour of the D_{13} state (N^{**}, 1518 MeV). In particular the S_{11} partial wave seems to be affected by the sharp opening of the rather abundant η production channel according to a cusp mechanism[6], as it was pointed out by Auvil et al.[2] and further investigated with different degree of success by several authors[7].

A rather convincing evidence has been found that the ηN system mainly passes through a $J = 1/2^-$, $T = 1/2$ resonance at the threshold. However some very recent phenomenological analyses[8] in which recent experimental results[9, 10] are taken into account show that a strong interaction in a S, P or D state cannot be undoubtely ruled out.

As far as the photoproduction reactions are concerned, due to the rather scanty experimental input available up to a quite recent past, a detailed multipole analysis comparable with the phase shift analysis of the πN scattering has not yet been done for energies much above the first resonance[11].

The situation around the second resonance is furthermore complicated by the presence of the known puzzle of the position in energy of the experimental maxima for the two reactions $\gamma p \rightarrow \pi^o p$ and $\gamma p \rightarrow \pi^+ n$. In regard to this particular aspect the possibility of a cusp effect of the η on the single pion photoproduction was already suggested by Rekalo(12). According to his prediction a subtractive anomaly could be expected in the differential cross section at 90^o for the $\gamma p \rightarrow \pi^o p$ reaction at the threshold for $\gamma p \rightarrow \eta\, p$ reaction.

On the other hand, a comparative analysis(9) has shown that, not too far from the η threshold, the behaviour with the energy of the production of η's and π's in $T = 1/2$ state and also the relative abundance of π's with respect to η's for both the initials states $\pi + N$ and $\gamma + N$ are practically the same. This seems to indicate that a cusp effect due to the η production is reasonably to be expected also in single photoproduction processes.

In this respect among the single pion photoproduction reactions the $\gamma + p \rightarrow \pi^o + p$ seems to be rather suitable to show the effect, due to the absence of the direct interaction term (present with the charged pions) and because it has an energy behaviour very similar to the energy behaviour of the $\pi N \rightarrow \pi N$ reaction ($T = 1/2$).

We mantain here the assumption that the quantum number of the state through which the η production occurs, both in π-production and in photoproduction, are $l = 0$, $J = 1/2$, $T = 1/2$; the multipole mainly involved in π^o photoproduction is then the electric dypole $E^{\pi^o}_{o+}$.

It must be noted that looking for an anomaly in the $\gamma p \rightarrow \pi^o p$ reaction at the η threshold, because of the steepness of the production cross section and the relative low values it reaches (factor five on the cross sections with respect to the $\gamma p \rightarrow \pi^o p$), an effect is reasonably expected to be strongly energy dependent and quite small in absolute value. Therefore for an experimental investigation high energy resolution and good statistics are simultaneously required. Up to now these requirements are reasonably satisfied only for some measurements of differential cross section at 90^o CMS(13, 14, 15).

Keeping this in mind we summarize in fig. 1 the experimental situation as far as the differential cross section for $\gamma p \rightarrow \pi^o p$ at 90^o CMS, in the energy region of the second resonances is concerned.

In fig. 1 the Frascati data(x) together with the Stanford ones are shown because these experiments have been performed with comparable resolution, $\approx \pm 10$ MeV.

At a first examination of the experimental results a sort of structure could be recognized in the region of the rise of the second resonance, that is just at the threshold of the η photoproduction channel.

(x) - Thanks are due to the authors of ref. (14) who kindly supplied us with the 90^o experimental results prior to publication.

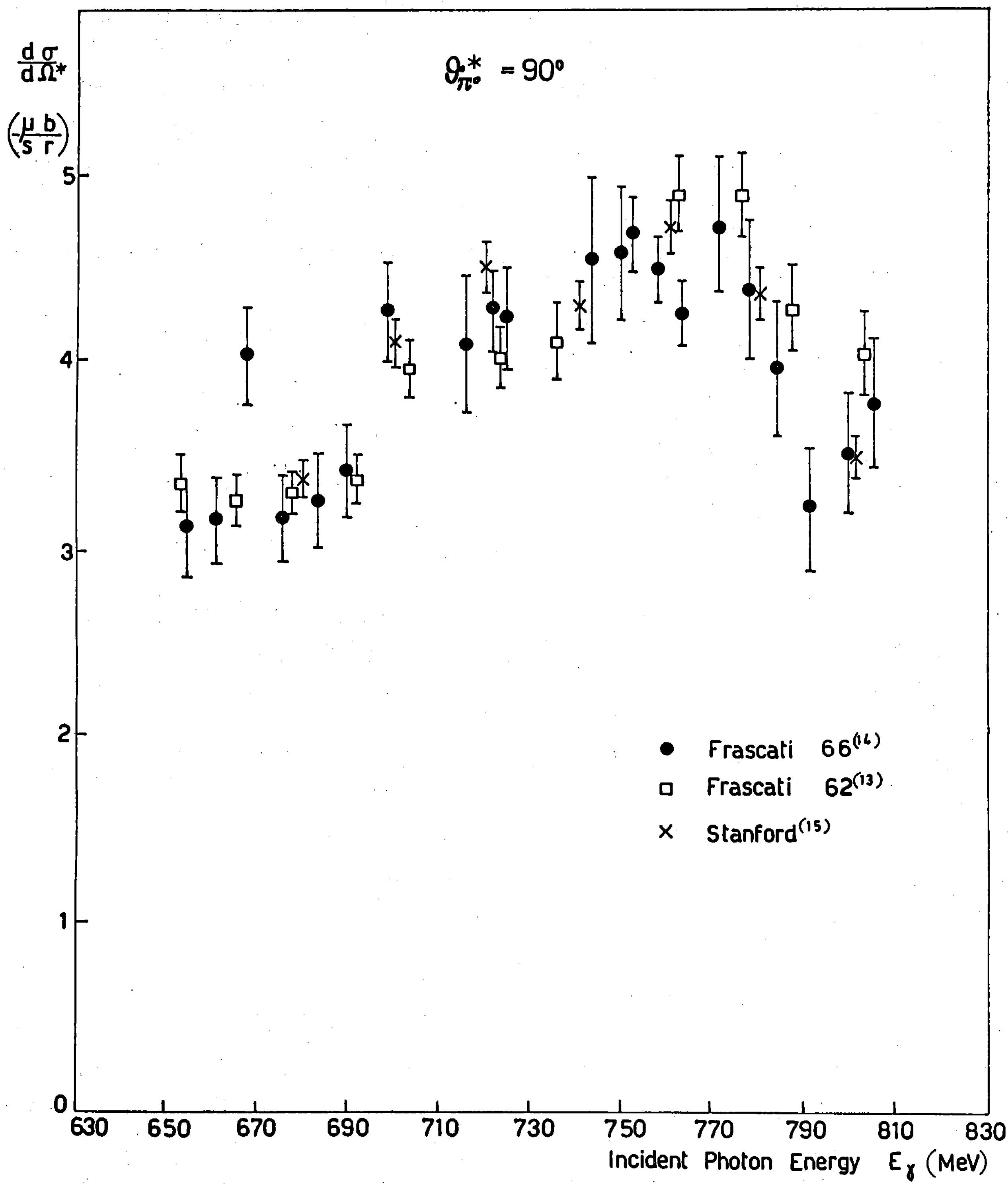

FIG. 1 - The differential cross section for π^0 photoproduction at $\theta^*_{\pi^0} = 90^0$, in the energy interval 650 - 810 MeV, as obtained in three experiments performed with good energy resolution.

No one of these measurements by itself gives a definite evidence of a clear structure, but all the results together are consistent with this indication.

A possible way through which the η channel opening could affect the π^0 photoproduction, at the threshold, is considered in the following.

In the energy region of the second resonance we consider the state $\ell = 0$, $J = 1/2$, $T = 1/2$, for the following reactions: $\pi N \longrightarrow \pi N$, $\pi N \longrightarrow \eta N$, $\gamma N \longrightarrow \pi N$, $\gamma N \longrightarrow \eta N$, $\eta N \longrightarrow \eta N$ and neglect the 2π production as the behaviour of the inelasticity parameter in the S_{11} partial wave seems to suggest(2 ÷ 5).

The use of the unitarity and time reversal invariance of the S-matrix leads to the following relationships:

$$\begin{aligned}
\langle \pi N | SS^+ | \pi N \rangle &= 1 \\
\langle \eta N | SS^+ | \pi N \rangle &= 0 \\
\langle \pi N | SS^+ | \gamma N \rangle &= 0 \\
\langle \eta N | SS^+ | \gamma N \rangle &= 0 \\
\langle \eta N | SS^+ | \eta N \rangle &= 1
\end{aligned}$$

If we put the S matrix elements in the form

$$\langle \pi | S | \pi \rangle = Ae^{2i\alpha}, \quad \langle \pi | S | \eta \rangle = Be^{i\beta}, \quad \langle \pi | S | \gamma \rangle = Ce^{i\gamma}, \langle \eta | S | \gamma \rangle = De^{i\delta},$$

$$\langle \eta | S | \eta \rangle = Fe^{2i\varphi}$$

then we get the following system of equations among different amplitudes and phases:

1) $A^2 + B^2 + C^2 = 1$

2) $AB\, e^{i(2\alpha - \beta)} + CDe^{i(\gamma - \delta)} + BF\, e^{i(\beta - 2\varphi)} = 0$

3) $AC\, e^{i(2\alpha - \gamma)} + C\, e^{i\gamma} + BD\, e^{i(\beta - \delta)} = 0$

4) $BC\, e^{i(\beta - \gamma)} + D\, e^{i\delta} + FD\, e^{i(2\varphi - \delta)} = 0$

5) $F^2 + D^2 + B^2 = 1$

By taking in account the relative order of magnitude of the different terms (according their "strong" or "e.m." nature) one can easily show that this system is equivalent to the simple equations:

$$C^2 = \frac{(1 - A^2)D^2}{1 + A^2 + 2\, A\cos 2(\alpha - \gamma)} \tag{6}$$

$$\frac{tg(\beta - \delta - \alpha)}{tg(\gamma - \alpha)} = \frac{1 - A}{1 + A}$$

We are interested in the transition amplitude $Ce^{i\gamma}$. The inelasticity parameter A and the phase shift α of the scattering are taken from ref.$^{(1 \div 4)}$.

The D amplitude can be determined from the knowledge of the $\gamma + p \rightarrow \eta + p$ photoproduction cross section which has been experimentally measured[16], according to[17]

$$\sigma_{Tot}\,(\gamma p \rightarrow \eta p) = \frac{\pi\,\lambda\!\!\!^{-2} D^2}{2}$$

where $\lambda\!\!\!^{-}$ is the wave length of the incident photon.

We now assume that both the reactions of photo and pion production of η's on nucleons are going through a resonant state of a Breit-Wigner type, that is $\pi N \rightarrow \Delta^* \rightarrow \eta N$, $\gamma N \rightarrow \Delta^* \rightarrow \eta N$; then we can reasonably believe that the energy behaviour is the same for the β and δ phases, because it is mainly determined from the energy and the width of the resonant state. We take, according to ref.[7] a value of $E\gamma \approx 750$ MeV for the resonance position in photoproduction.

We can then solve equation eq. (6) for C and γ by putting $\beta \approx \delta$ in a small region around 750 MeV.

We can also take profit of the fact that below the η threshold the scattering phase α and the transition phase $\gamma_o = \gamma - \pi/2$ (the $\pi/2$ term is a consequence of our definition of the transition elements) are equals[18] so that γ_o is completely determined everywhere.

A graph of γ_o is given in fig. 2.

Concerning the C amplitude let us recall that the total $E^{\pi^o}_{o+}$ photoproduction amplitude is equal to

$$E^{\pi^o}_{o+} = \frac{2}{3} E^{3/2}_{o+} + \frac{1}{3} (E^{1/2}_{o+} + 3\, E^{o}_{o+})$$

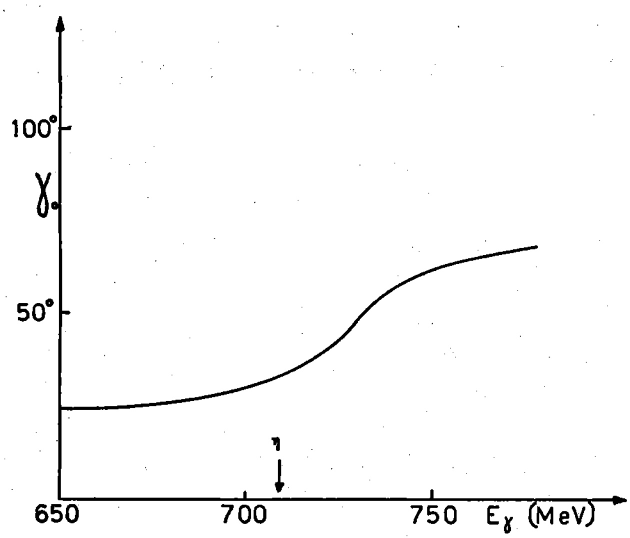

FIG. 2 - The phase of the S-matrix element for $\gamma p \rightarrow \pi p$ in the $T = 1/2$, $J = 1/2^-$ state vs the energy of the incident γ (LS).

$E^{3/2}_{o+}$, $E^{1/2}_{o+}$, E^{o}_{o+} are the 3/2, 1/2 isovector parts and the isoscalar part respectively(x).

For the energies below the η threshold, we take for $E^{\pi o}_{o+}$ the values which can be extrapoled from ref.(11), while in the ηN resonance region we have

$$E^{\pi o}_{o+} = \frac{2}{3} E^{3/2}_{o+} + \frac{1}{3} (C\, e^{i\gamma o})$$

where only the $E^{3/2}_{o+}$ part (which is not expected to have a fast changing behaviour in our energy region) has been extrapolated from ref.(11).

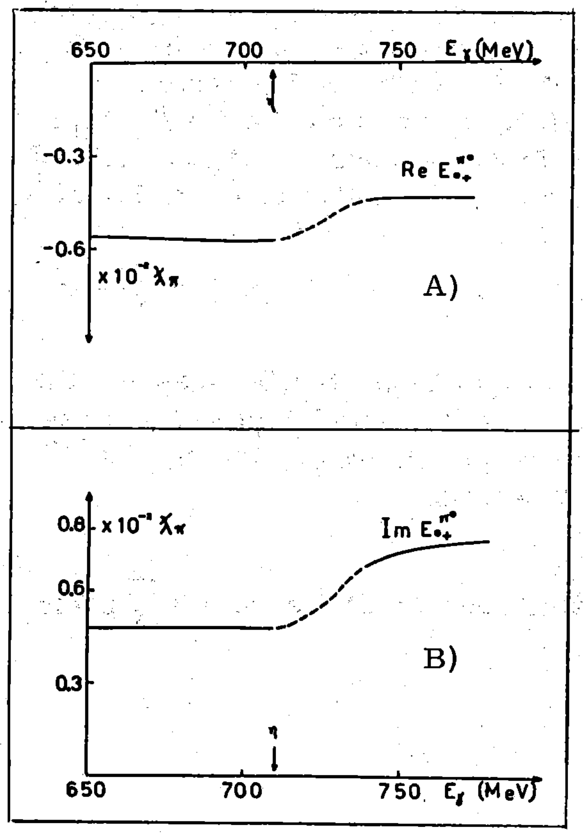

FIG. 3 - The isospin 1/2 E_{o+} multipole for $\gamma p \rightarrow \pi p$ vs the energy of the incident γ (L S).

The real and immaginary parts of $E^{\pi o}_{o+}$ are given in fig. 3A, B respectively.

To get the effect on the cross section one should know the behaviour of the multipoles which are effective in the region of the second resonance.

In fact, if for instance the assumption is made that only the multipoles E_{o+}, M_{1+}, E_{2-}, M_{2-} are important, the 90° differential cross section can be written:

$$\frac{K\, d\sigma}{q\, d\Omega^*} = \left|E_{o+}\right|^2 + \frac{5}{2}\left|M_{1+}\right|^2 + \tag{7}$$
$$+\frac{5}{2}\left|E_{2-}\right|^2 + \frac{9}{2}\left|M_{2-}\right|^2 - \mathrm{Re}\,\cdot$$
$$\cdot\left[E^*_{o+}(E_{2-} - 3M_{2-})\right] + 3\mathrm{Re}\,M^*_{2-}E_{2-}$$

where K, q are the CM momenta of the photon and the meson respectively.

Unfortunately the knowledge of the multipoles is as yet not complete in this region and any attempt to make a quantitative estimate of the η

(x) - Because of the Watson theorem, below the η threshold

$$\mathrm{Im}E^{3/2}_{o+} = \mathrm{Re}E^{3/2}_{o+}\, tg\,\alpha_{31}; \qquad \mathrm{Im}(E^{1/2}_{o+} + 3E^{o}_{o+}) = \mathrm{Re}(E^{1/2}_{o+} + 3E^{o}_{o+})\, tg\,\alpha_{11}$$

where α_{31} and α_{11} ($\equiv\alpha$) are the phase shifts for the 3/2 and 1/2 isotopic spin states in πN scattering.

anomaly on the cross section at the threshold, according to (7), is somewhat dependent on the model used for the multipoles involved.

We report elsewhere[14] in a more extensive form, the details of the calculations performed to fit by (7) the experimental results of fig. 1. Additional informations on the others multipoles necessary to build up the cross section have been taken from ref.[11]. We just note here that the calculated anomaly is much smaller than one can argue from the experimental distribution: the perturbation on the $E_{0+}^{\pi^0}$ multipole due to the sharp opening of the η channel is not great enough to account for the shoulder observed at $E\gamma$ = 700 - 740 MeV.

On the other hand the remark has to be done that our model for the ηN system could be an oversimplified one. In fact the possibility that the ηN resonance could be a P_{11} state or even a mixture of states cannot be ruled out by the present experimental evidence.

We wish to thank prof. G. Salvini for the frequent and valuable discussions on the problem and profs. M. Bassetti and C. Pellegrini for their helpful remarks.

Prof. W. Schmidt is also gratefully aknowledged for his criticism.

Our thanks are also due to the computer staff and mainly to dr. G. Trenta.

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